THEORY OF CIRCULAR DIELECTRIC WAVEGUIDE WITH ANISOTROPIC SHEET COVER

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THESIS

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SUMMARY

A circular dielectric waveguide consisting of an isotropic core covered with a thin anisotropic sheet is considered. The sheet is represented as a jump immittance and Maxwell's equations are applied. Solution of the boundary value problem yields the characteristic equation, or dispersion relation, which is then solved numerically. The results are verified for the step-index fiber and circular, metallic waveguides. Finally, examples are included to investigate the effects of the anisotropic sheet.

1. INTRODUCTION

1.1 Background

The solution to the problem of wave propagation in an isotropic, circularly symmetric optical waveguide is covered in many texts. In particular, each of references [1]-[3] provides an analysis of the step-index optical fiber. The step-index fiber is treated as a boundary-value problem which is solved by use of electromagnetic field theory and the separation of variables technique. The result is the characteristic equation of the guide and its solutions yield the propagation constants of the modes existing in the waveguide.

Recently, efforts have been made to find modal solutions for similar structures composed of anisotropic materials for applications in polarization maintaining optical fibers. Related structures composed of metallic waveguides coated with anisotropic layers have also been analyzed for radar cross section reduction purposes. In particular, Kapany and Burke [4] describe a circularly cylindrical waveguide with an isotropic cladding and an anisotropic core whose optical axis coincides with the axis of the cylinder. Tonning [5] reformulates the problem using a matrix formalism, and then extends the work of Kapany and Burke by examining a waveguide with a uniaxial core as well as cladding [6]. Another alternative approach is the variational analysis provided by Lindell and Oksanen [7] for the case of transverse anisotropy. Using this method, they compare their results to the exact eigenvalue equation for a metallic, circular guide coated with a layer characterized by an isotropic surface impedance [8]. For an anisotropic surface impedance, solutions have been obtained for propagation in the circular metallic guide [9] . Chou and Lee analyze the propagation and attenuation characteristics of a multilayered coated waveguide and then examine the radar cross section of the waveguide at near axial incidence [10]. Scattering from a multilayered cylindrical structure was studied by Graglia and Uslenghi [11] with each layer being represented by an anisotropic jump impedance. Thus there are a variety of methods available to analyze anisotropic structures.

1.1 Outline of Research

Using the jump immittance model, modal solutions will be obtained for a structure consisting of an isotropic, dielectric core covered with a thin anisotropic sheet surrounded by free space. The anisotropic sheet is modeled as a sheet of zero thickness with a jump admittance. The jump immittance model is convenient to implement and describes the electromagnetic behavior of a thin layer of penetrable material [12]. It is, therefore, possible to impose the appropriate boundary conditions and complete a field analysis using Maxwell's equations. The derivation follows that of the step-index optical fiber boundary value problem. The only difference is in the application of the boundary conditions as dictated by the jump immitance condition. The resulting dispersion relation, or eigenvalue equation, given in Section 2.3 describes the mode structure of the guide. Also derived are the cutoff conditions of the various modes (Section 2.4).

Since the jump admittance of the anisotropic sheet is generally complex in value, the dispersion relation and cutoff conditions are functions of complex variables. The solutions of these equations are the propagation and attenuation constants for the various modes that exist in the guide. As shown in Chapter 2, for limiting cases of the jump admittance, the dispersion relation and cutoff conditions approach the solutions for the isotropic step-index fiber and circular,metallic guide. To find the roots of complex-valued equations, the Newton-Raphson iterative procedure was employed [13]. The FORTRAN program constructed to apply the Newton-Raphson technique to the dispersion relation is given in Appendix A.

Chapter 3 provides the results of the numerical solution of the dispersion relation. First, the solutions are tested against known results for the step-index fiber and circular metallic guide. Next, cutoff values for the propagation constant are computed for the case of a lossless sheet. Finally, examples are included to illustrate how propagation and attenuation of the modes of the waveguide are affected by variations in the parameters of the anisotropic sheet.

1.3 Geometry of Waveguide

The cross section of the circularly symmetric waveguide is shown in Figure 1. A dielectric core of radius $\rho = a$ and index of refraction $n = \sqrt{\epsilon_r \mu_r}$ is coated by an anisotropic layer.

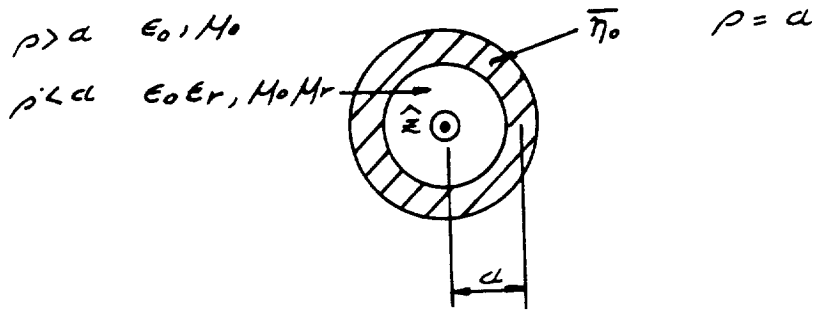

Figure 1. Waveguide cross section.

The layer is modeled with a jump admittance given by [11]:

$$\bar{\eta}_0 = Y_0 \begin{pmatrix} \eta_{01} & 0 \\ 0 & \eta_{02} \end{pmatrix}_{\zeta,\xi} \quad ; \quad Y_0 = \sqrt{\epsilon_0/\mu_0} \qquad (1)$$

The coordinates of the sheet and rod are shown in Figure 2.

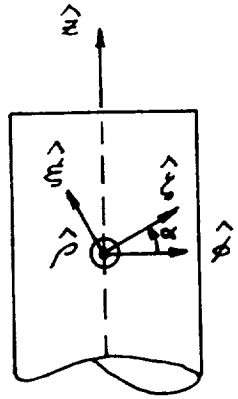

Figure 2. Orientation of coordinate axes.

A coordinate transformation [14] is performed represented by:

$$\bar{\eta} = \mathbf{R}\bar{\eta}_0\mathbf{R}^{\mathbf{T}}, \tag{2a}$$

where

$$\mathbf{R} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}. \tag{2b}$$

This transforms the arbitrary orientation of the sheet admittance into the ρ, ϕ, z coordinates of the rod. The result is:

$$\bar{\eta} = Y_0 \begin{pmatrix} \eta_1 & \eta_3 \\ \eta_3 & \eta_2 \end{pmatrix}_{\phi,z} \tag{3}$$

with

$$\eta_1 = \eta_{01}\cos^2\alpha + \eta_{02}\sin^2\alpha \tag{4a}$$

$$\eta_2 = \eta_{01}\sin^2\alpha + \eta_{02}\cos^2\alpha \tag{4b}$$

$$\eta_3 = \sin\alpha\cos\alpha(\eta_{01} - \eta_{02}) \tag{4c}$$

2. FIELD SOLUTION

2.1 Field Components

Maxwell's curl equations for fields of angular frequency ω are, in the absence of sources:

$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E} \tag{5a}$$

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}. \tag{5b}$$

We consider modes with longitudinal wave number β and attempt solutions of the form:

$$\mathbf{F} = \mathbf{F}(\rho)e^{-j\beta z + jm\phi + j\omega t} \tag{6}$$

where $\mathbf{F}$ represents either the $\mathbf{E}$ or $\mathbf{H}$ field vector [6]. Note that

$$\frac{\partial \mathbf{F}}{\partial z} = -j\beta \tag{7}$$

Also, the periodicity of 2π radians in the variable ϕ requires that

$$\frac{\partial \mathbf{F}}{\partial \phi} = jm; \qquad m = 0, \pm 1, \pm 2, \cdots \tag{8}$$

The curls of (5) are expressed in the given cylindrical coordinates to produce a set of six scalar equations. With the use of (7) and (8), the transverse components E_ϕ, E_ρ, H_ϕ , and H_ρ may be expressed as functions of the longitudinal components E_z and H_z , as follows:

$$E_\rho = \frac{1}{k_t^2}\left(-j\beta\frac{dE_z}{d\rho} + \frac{m\omega\mu}{\rho}H_z\right) \tag{9a}$$

$$E_\phi = \frac{1}{k_t^2}\left(\frac{m\beta}{\rho}E_z + j\omega\mu\frac{dH_z}{d\rho}\right) \tag{9b}$$

$$H_\rho = \frac{1}{k_t^2}\left(\frac{-m\omega\epsilon}{\rho}E_z - j\beta\frac{dH_z}{d\rho}\right) \tag{9c}$$

$$H_\phi = \frac{1}{k_t^2}\left(-j\omega\epsilon\frac{dE_z}{d\rho} + \frac{m\beta}{\rho}H_z\right). \tag{9d}$$

The transverse wave number k_t is given by

(10a) $$k_t^2 = k^2 - \beta^2$$

(10b) $$k = \begin{cases} \omega\sqrt{\epsilon\mu} = k_0 n, & \rho < a \\ \omega\sqrt{\epsilon_0\mu_0} = k_0, & \rho > a \end{cases}.$$

For the region inside the core we must establish fields E_z and H_z which are finite at $\rho = 0$.

Choosing

(11) $$k_t^2 = k_0^2 n^2 - \beta^2,$$

we have for $\rho < a$

(12) $$F_z \propto J_m(k_t\rho)e^{jm\phi - j\beta z}.$$

Outside the core, an evanescent field is required,or

(13) $$\begin{aligned} k_{0t}^2 &= k_0^2 - \beta^2 \\ &= -\gamma_t^2 < 0. \end{aligned}$$

We have for $\rho > a$

(14) $$F_z \propto K_m(\gamma_t\rho)e^{jm\phi - j\beta z},$$

where K_m is the modified Bessel function of the second type. This guarantees exponential decay as $\rho \to \infty$. Also, both k_t^2 and γ_t^2 must be greater than zero for guidance. The bounds on β are then

(15) $$k_0 n > \beta > k_0 \quad .$$

The resulting modal components of all fields are:

in the core, $\rho < a$:

$$E_z = AJ_m(k_t\rho)e^{jm\phi - j\beta z} \tag{16a}$$

$$H_z = BJ_m(k_t\rho)e^{jm\phi - j\beta z} \tag{16b}$$

$$E_\rho = [\frac{-j\beta}{k_t}AJ'_m(k_t\rho) + \frac{m\omega\mu}{k_t^2\rho}BJ_m(k_t\rho)]e^{jm\phi - j\beta z} \tag{16c}$$

$$E_\phi = [\frac{m\beta}{k_t^2\rho}AJ_m(k_t\rho) - \frac{j\beta}{k_t}BJ'_m(k_t\rho)]e^{jm\phi - j\beta z} \tag{16d}$$

$$H_\rho = [\frac{-m\omega\epsilon}{k_t^2\rho}AJ_m(k_t\rho) - \frac{j\beta}{k_t}BJ'_m(k_t\rho)]e^{jm\phi - j\beta z} \tag{16e}$$

$$H_\phi = [\frac{-j\omega\epsilon}{k_t}AJ'_m(k_t\rho) + \frac{m\beta}{k_t^2\rho}BJ_m(k_t\rho)]e^{jm\phi - j\beta z}, \tag{16f}$$

and outside the core, $\rho > a$:

$$E_z = CK_m(\gamma_t\rho)e^{jm\phi - j\beta z} \tag{17a}$$

$$H_z = DK_m(\gamma_t\rho)e^{jm\phi - j\beta z} \tag{17b}$$

$$E_\rho = [\frac{-\beta}{j\gamma_t}CK'_m(\gamma_t\rho) - \frac{m\omega\mu_0}{\gamma_t^2\rho}DK_m(\gamma_t\rho)]e^{jm\phi - j\beta z} \tag{17c}$$

$$E_\phi = [-\frac{m\beta}{\gamma_t^2\rho}CK_m(\gamma_t\rho) + \frac{\omega\mu_0}{j\gamma_t}DK'_m(\gamma_t\rho)]e^{jm\phi - j\beta z} \tag{17d}$$

$$H_\rho = [\frac{m\omega\epsilon_0}{\gamma_t^2\rho}CK_m(\gamma_t\rho) - \frac{\beta}{j\gamma_t}DK'_m(\gamma_t\rho)]e^{jm\phi - j\beta z} \tag{17e}$$

$$H_\phi = [-\frac{\omega\epsilon_0}{j\gamma_t}CK'_m(\gamma_t\rho) - \frac{m\beta}{\gamma_t^2\rho}DK_m(\gamma_t\rho)]e^{jm\phi - j\beta z} \tag{17f}$$

2.2 Application of Boundary Conditions

We have already satisfied the boundary conditions at $\rho = 0$ and as $\rho \to \infty$ as described by (12) and (14). Across the interface at $\rho = a$, the tangential electric field is continuous:

$$(E_z)_{\rho=a_-} = (E_z)_{\rho=a_+} \tag{18a}$$

$$(E_\phi)_{\rho=a_-} = (E_\phi)_{\rho=a_+}. \tag{18b}$$

The tangential magnetic field is discontinuous and satisfies the jump condition [10]:

$$\hat{\rho} \times (\mathbf{H}_{\rho=a_+} - \mathbf{H}_{\rho=a+-}) = \overline{\eta}(\mathbf{E}_{\tan})_{\rho=a} \quad , \tag{19}$$

or,

$$-H_z(a_+) + H_z(a_-) = Y_0[\eta_1 E_\phi(a) + \eta_3 E_z(a)] \tag{20a}$$

$$H_\phi(a_+) - H_\phi(a_-) = Y_0[\eta_3 E_\phi(a) + \eta_2 E_z(a)]. \tag{20b}$$

Equations (18) and (20) will be applied to the field equations of (16) and (17). The result is a homogeneous system of four equations in the four unknowns A, B, C,and D,

$$\mathbf{M} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \mathbf{0}. \tag{21}$$

For a nontrivial solution it is necessary that $\det\mathbf{M} = 0$. The resulting equation is known as the dispersion relation.

2.3 Formulation of Dispersion Relation

As shown in (21), the coefficients A, B, C,and D,are chosen as the column vector and (18) and (20) are the rows. The right hand sides of (20) are evaluated at $\rho = a_-$. The matrix $\mathbf{M}$ becomes:

$$\mathbf{M} = \begin{pmatrix} J_m(k_t a) & 0 & -K_m(\gamma_t a) & 0 \\ \frac{m\beta}{k_t^2 a} J_m(k_t a) & \frac{j\omega\mu}{k_t} J'_m(k_t a) & \frac{m\beta}{\gamma_t^2 a} K_m(\gamma_t a) & \frac{-\omega\mu_0}{j\gamma_t} K'_m(\gamma_t a) \\ -Y_0 J_m(k_t a)(\eta_3 + \eta_1 \frac{m\beta}{k_t^2 a}) & J_m(k_t a) - Y_0 \eta_1 \frac{j\omega\mu}{k_t} J'_m(k_t a) & 0 & -K_m(\gamma_t a) \\ \frac{-j\omega\epsilon}{k_t} J'_m(k_t a) + Y_0(\eta_2 + \eta_3 \frac{m\beta}{k_t^2 a}) J_m(k_t a) & \frac{m\beta}{k_t^2 a} J_m(k_t a) + Y_0 \eta_3 \frac{j\omega\mu}{k_t} J'_m(k_t a) & \frac{\omega\epsilon_0}{j\gamma_t} K'_m(\gamma_t a) & \frac{m\beta}{\gamma_t^2 a} K_m(\gamma_t a) \end{pmatrix} \tag{22}$$

Taking the determinant of (22):

$$J_m(k_t a) \begin{vmatrix} \frac{j\omega\mu}{k_{t0}} J'_m(k_t a) & \frac{m\beta}{\gamma_t^2 a} K_m(\gamma_t a) & -\frac{\omega\mu_0}{j\gamma_t} K'_m(\gamma_t a) \\ J_m(k_t a) - Y_0 \eta_1 \frac{j\omega\mu}{k_t} J'_m(k_t a) & 0 & -K_m(\gamma_t a) \\ \frac{m\beta}{k_t^2 a} J_m(k_t a) + Y_0 \eta_3 \frac{j\omega\mu}{k_t} J'_m(k_t a) & \frac{\omega\epsilon_0}{j\gamma_t} K'_m(\gamma_t a) & \frac{m\beta}{\gamma_t^2 a} K_m(\gamma_t a) \end{vmatrix}$$

$$-K_m(\gamma_t a) \begin{vmatrix} \frac{m\beta}{k_t^2 a} J_m(k_t a) & \frac{j\omega\mu}{k_t} J'_m(k_t a) & \frac{-\omega\mu_0}{j\gamma_t} K'_m(\gamma_t a) \\ -Y_0 J_m(k_t a)(\eta_3 + \eta_1 \frac{m\beta}{k_t^2 a}) & J_m(k_t a) - Y_0 \eta_1 \frac{j\omega\mu}{k_t} J'_m(k_t a) & -K_m(\gamma_t a) \\ \frac{-j\omega\epsilon}{k_t} J'_m(k_t a) + Y_0(\eta_2 + \eta_3 \frac{m\beta}{k_t^2 a}) J_m(k_t a) & \frac{m\beta}{k_t^2 a} J_m(k_t a) + Y_0 \eta_3 \frac{j\omega\mu}{k_t} J'_m(k_t a) & \frac{m\beta}{\gamma_t^2 a} K_m(\gamma_t a) \end{vmatrix} = 0. \tag{23}$$

This last expression leads to the following result:

$$\frac{n^2k_0^2}{k_t^2}\left(\frac{J_m'(k_ta)}{k_tJ_m(k_ta)}\right)^2+\frac{jk_0^2}{k_t\gamma_t}\frac{J_m'(k_ta)}{k_tJ_m(k_ta)}\frac{K_m'(\gamma_ta)}{j\gamma_tK_m(\gamma_ta)}(\epsilon/\epsilon_0+\mu/\mu_0)$$

$$-\frac{k_0^2}{\gamma_t^2}\left(\frac{K_m'(\gamma_ta)}{j\gamma_tK_m(\gamma_ta)}\right)^2-\left(\frac{m\beta}{a}\right)^2\left(\frac{1}{\gamma_t^2}+\frac{1}{k_t^2}\right)^2$$

$$+\frac{j\omega^2\mu^2\mu_r}{k_t\gamma_t}Y_0^2\frac{J_m'(k_ta)}{k_tJ_m(k_ta)}\frac{K_m'(\gamma_ta)}{j\gamma_tK_m(\gamma_ta)}(\eta_1\eta_2-\eta_3^2)$$

$$+\frac{\omega}{\gamma_tk_t}Y_0\eta_1\frac{J_m'(k_ta)}{k_tJ_m(k_ta)}\frac{K_m'(\gamma_ta)}{j\gamma_tK_m(\gamma_ta)}\left(\frac{\mu_0n^2k_0^2}{k_t}\frac{J_m'(k_ta)}{k_tJ_m(k_ta)}+\frac{j\mu k_0^2}{\gamma_t}\frac{K_m'(\gamma_ta)}{j\gamma_tK_m(\gamma_ta)}\right)$$

$$+\frac{j\omega\mu}{k_t}Y_0\frac{J_m'(k_ta)}{k_tJ_m(k_ta)}[\left(\frac{m\beta_0}{\gamma_t^2a}\right)^2\eta_1-2\frac{m\beta_0}{\gamma_t^2a}\eta_3+\eta_2]$$

$$-\frac{\omega\mu_0}{\gamma_t}Y_0\frac{K_m'(\gamma_ta)}{j\gamma_tK_m(\gamma_ta)}[\left(\frac{m\beta_0}{k_t^2a}\right)^2\eta_1+2\frac{m\beta_0}{k_t^2a}\eta_3+\eta_2]=0. \tag{24}$$

To develop a general case, normalized variables [4] are introduced according to the following equations:

$$b_0=\beta/k_0 \tag{25a}$$

$$k_{t0}=k_t/k_0 \tag{25b}$$

$$\gamma_{t0}=\gamma_t/k_0 \tag{25c}$$

$$a_0=ak_0. \tag{25d}$$

The guidance condition becomes:

$$n>b_0>1. \tag{26}$$

Equation (24) becomes:

$$\epsilon_r\mu_r\left(\frac{J_m'(k_{t0}a_0)}{k_{t0}J_m(k_{t0}a_0)}\right)^2+\frac{J_m'(k_{t0}a_0)}{k_{t0}J_m(k_{t0}a_0)}\frac{K_m'(\gamma_{t0}a_0)}{\gamma_{t0}K_m(\gamma_{t0}a_0)}(\epsilon_r+\mu_r)$$

$$+\left(\frac{K_m'(\gamma_{t0}a_0)}{\gamma_{t0}K_m(\gamma_{t0}a_0)}\right)^2-\left(\frac{mb_0}{a_0}\right)^2\left(\frac{1}{\gamma_{t0}^2}+\frac{1}{k_{t0}^2}\right)^2$$

$$+\mu_r\frac{J_m'(k_{t0}a_0)}{k_{t0}J_m(k_{t0}a_0)}\frac{K_m'(\gamma_{t0}a_0)}{\gamma_{t0}K_m(\gamma_{t0}a_0)}(\eta_1\eta_2-\eta_3^2)$$

$$-j\eta_1\mu_r\frac{J_m'(k_{t0}a_0)}{k_{t0}J_m(k_{t0}a_0)}\frac{K_m'(\gamma_{t0}a_0)}{\gamma_{t0}K_m(\gamma_{t0}a_0)}\left(\epsilon_r\frac{J_m'(k_{t0}a_0)}{k_{t0}J_m(k_{t0}a_0)}+\frac{K_m'(\gamma_{t0}a_0)}{\gamma_{t0}K_m(\gamma_{t0}a_0)}\right)$$

$$+j\mu_r\frac{J_m'(k_{t0}a_0)}{k_{t0}J_m(k_{t0}a_0)}\left[\left(\frac{mb_0}{\gamma_{t0}^2a_0}\right)^2\eta_1-2\frac{mb_0}{\gamma_{t0}^2a_0}\eta_3+\eta_2\right]$$

$$+j\frac{K_m'(\gamma_{t0}a_0)}{\gamma_{t0}K_m(\gamma_{t0}a_0)}\left[\left(\frac{mb_0}{k_{t0}^2a_0}\right)^2\eta_1+2\frac{mb_0}{k_{t0}^2a_0}\eta_3+\eta_2\right]=0. \tag{27}$$

To write more compactly, let

$$u=k_{t0}a_0 \tag{28a}$$

$$q=\gamma_{t0}a_0 \tag{28b}$$

$$\mathcal{J}_m=\frac{J_m'(u)}{uJ_m(u)} \tag{28c}$$

$$\mathcal{K}_m=\frac{K_m'(q)}{qK_m(q)} \tag{28d}$$

Note that the dispersion relation is quadratic in $\mathcal{J}_m$. It is of the form

$$C_0\mathcal{J}_m^2+C_1\mathcal{J}_m+C_2=0 \tag{29}$$

Collecting terms,

$$\begin{aligned}
&\mathcal{J}_m^2\epsilon_r\mu_r(1-j\eta_1 a_0\mathcal{K}_m)\\
&+\mathcal{J}_m\left\{-j\eta_1\mu_r a_0\mathcal{K}_m^2+\mathcal{K}_m[\mu_r(1+\eta_1\eta_2-\eta_3^2)+\epsilon_r]\right.\\
&\qquad\left.+\frac{j\mu_r}{a_0}\left[\left(\frac{mb_0a_0}{q^2}\right)^2\eta_1-2\frac{mb_0a_0}{q^2}\eta_3+\eta_2\right]\right\}\\
&+\mathcal{K}_m^2+\frac{j\mathcal{K}_m}{a_0}\left[\left(\frac{mb_0a_0}{u^2}\right)^2\eta_1+2\frac{mb_0a_0}{u^2}\eta_3+\eta_2\right]-(mb_0)^2\left(\frac{1}{u^2}+\frac{1}{q^2}\right)^2=0 \quad . \qquad (30)
\end{aligned}$$

The step-index solution is contained in the real portion of (30) , while the anisotropic sheet adds additional complex terms. In this form, the dispersion relation is solved numerically; its roots are computed as explained in Appendix A. The solutions are [4]:

$$\text{EH}_{mn}\text{ modes}: \quad \mathcal{J}_m=\frac{-C_1+\sqrt{C_1^2-4C_0C_2}}{2C_0} \qquad (31a)$$

$$\text{HE}_{mn}\text{ modes}: \quad \mathcal{J}_m=\frac{-C_1-\sqrt{C_1^2-4C_0C_2}}{2C_0} \quad . \qquad (31b)$$

Two cases are examined to test the jump admittance condition. The relative sheet admittances are complex:

$$\eta_i=g_i+jb_i\begin{cases} g_i,b_i & \text{real,with:}\\ g_i=0 & \text{lossless sheets,}\\ g_i=b_i=0 & \text{no sheet present,}\\ 1/b_i=0 & \text{perfectly conducting sheet.}\end{cases} \qquad (32)$$

Substituting the condition for no sheet, $\eta_i=0$, (30) obviously reduces to the equation for the step-index guide [1],

$$\epsilon_r\mu_r\mathcal{J}_m^2+\mathcal{J}_m\mathcal{K}_m(\epsilon_r+\mu_r)-(mb_0)^2\left(\frac{1}{u^2}+\frac{1}{q^2}\right)^2=0 \quad . \qquad (33)$$

Next, the metallic boundary of a perfect conductor is a particular case of an isotropic surface admittance ($\eta_3=0$) in the limit as $\eta_1=\eta_2\to\infty$. If (30) is multiplied by the factor $\frac{J_m^2(u)}{\eta_1\eta_2}$, the defining equation for the circular, metallic guide results:

$$J_m'(u)J_m(u)=0 \quad . \qquad (34)$$

At the limiting cases of no sheet and a perfectly conducting sheet, the derived dispersion relation matches the dispersion relations for the step-index fiber and the circular metallic guide.

2.4 Derivation of Cutoff Conditions

2.4.1 Limits at Cutoff

To find the cutoff conditions of the various modes, (30) is examined in the limit as

$$q \to 0 \tag{35a}$$

$$b \to 1 \tag{35b}$$

$$u \to a_0\sqrt{\epsilon_r\mu_r - 1} \quad . \tag{35c}$$

Under these conditions, $\mathcal{K}_m$ takes the following form [4]:

$$m = 0; \quad \mathcal{K}_0 \to \frac{1}{q^2 ln\frac{\Gamma q}{2}} \tag{36a}$$

$$m = 1; \quad \mathcal{K}_1 \to -\frac{1}{q^2} + \frac{1}{2} ln\frac{q}{2} \tag{36b}$$

$$m > 1; \quad \mathcal{K}_m \to -\frac{m}{q^2} - \frac{1}{2(m-1)}. \tag{36c}$$

A detailed derivation of the cutoff conditions is presented in Appendix B.

2.4.2 Case I. : $m > 1$ At first glance, the dispersion relation would appear to be a function of $\frac{1}{q^2}$ given the order of (36c). Upon combining the higher order terms and using the relation $q^2 = a_0^2(b_0^2 - 1)$, it is seen that $\frac{1}{q^2}$ is the highest factor. Equation (30) is multiplied by the factor $\frac{q^2}{m}$ with the following result:

$$\begin{aligned} &\mathcal{J}_m^2 j\eta_1\epsilon_r\mu_r a_0 + \\ &\mathcal{J}_m\left[j\eta_1\mu_r\left(\frac{m}{a_0} - \frac{a_0}{m-1}\right) - 2j\eta_3\mu_r - \mu_r(1+\eta_1\eta_2 - \eta_3^2) - \epsilon_r\right] \\ &-\frac{1}{a_0}\left(\frac{m}{a_0} - \frac{a_0}{m-1}\right) - \frac{2m}{u^2} - \frac{j}{a_0}\left((\frac{ma_0}{u^2})^2\eta_1 + 2\frac{ma_0}{u^2}\eta_3 + \eta_2\right) = 0 \quad . \end{aligned} \tag{37}$$

A numerical solution is computed from an equation of this form. The cutoff conditions are defined for modes in (31) with $m > 1$.

2.4.3 Case II. : $m = 1$ The $m = 1$ cutoff conditions are found using K_1 as defined by (36b). After performing a similar simplifying combination as in Section 2.4.2, the dispersion relation is multiplied by $J_m(u)\frac{a^2}{ln\frac{a}{b}}$. The result is

$$J_1(u)[-j\eta_1\mu_r a_0 J_1'(u) + J_1(u)] = 0 \quad , \tag{38}$$

which implies the cutoff conditions:

$$\mathrm{EH}_{1n} \text{ modes}: \quad J_1(u) = 0 \qquad a_0 \neq 0 \tag{39a}$$

$$\mathrm{HE}_{1n} \text{modes}: \quad \frac{J_1'(u)}{uJ_1(u)} = \frac{1}{j\eta_1\mu_r a_0} \quad . \tag{39b}$$

2.4.4 Case III. : $m = 0$ For modes of the order $m = 0$,the dispersion relation factors easily. The resulting cutoff conditions from Appendix B are:

$$\mathrm{TM}_{0n} \text{ modes}: \quad J_0(u) = 0 \tag{40a}$$

$$\mathrm{TE}_{0n} \text{ modes}: \quad \frac{J_0'(u)}{uJ_0(u)} = \frac{\epsilon_r - \eta_3^2}{j\eta_1 a_0 \epsilon_r \mu_r} \quad . \tag{40b}$$

Since the cutoff conditions (37),(39), and (40) are a special case of the dispersion relation, it is not surprising that they also produce the expected cutoff equations when examined for the limiting cases of no sheet and a perfectly conducting sheet. Note that the EH and HE modes of the step-index fiber become respectively the corresponding order TM and TE modes of the metallic guide [10]. For $m \leq 1$, the TM and EH mode cutoff remains unchanged regardless of the lossless sheet parameters. The equation determining the cutoff frequencies becomes more complicated for higher order modes.

Given that the normalized propagation constant of (35) is real, the cutoff conditions are defined for propagating modes which are lossless. Therefore, sheet parameters contained in the preceding cutoff equations are taken as pure imaginary. Simple solutions for lossy sheets are difficult to obtain. However, such solutions are easily computed due to the availability of subroutines which calculate Bessel functions of complex arguments.

3. NUMERICAL RESULTS AND CONCLUSION

3.1 Lossless Sheet Parameters

Due to the complexity of the dispersion relation, a numerical solution is attempted. Appendix A outlines the solution and provides the FORTRAN program constructed for the purpose of obtaining the zeros of the dispersion relation. Lossless examples are examined first; the sheet admittance values are chosen to be pure imaginary. Marcuse [2] examines a step-index guide with $\epsilon_r = 1.0201$ and $\mu_r = 1.0$, plotting normalized propagation constant against normalized radius. The results of Figure 3 agree with those shown in [2]. The HE_{11} mode is actually cutoff at zero, but the curve is nearly parallel to the a_0 axis at $a_0 = 5$. The HE_{21} mode is also plotted and it is not discernible from the TE_{01} and TM_{01} curve.

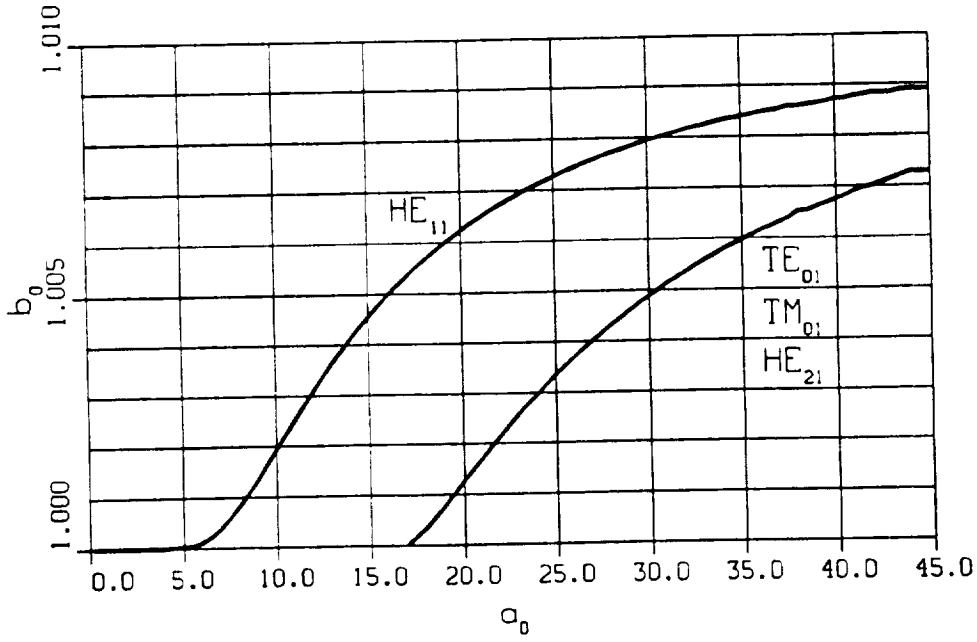

Figure 3. Normalized propagation constant vs. normalized radius for step-index fiber.

In Figure 4, plots of the step-index guide are provided with those of the corresponding modes of the circular, metallic guide. Here, the horizontal axis has been changed to the cutoff number $V = a_0\sqrt{\epsilon_r \mu_r - 1}$. Remember, the TM_{01} cutoff value remains unchanged. In genereal, as η_{01} varies from $-j0$ to $-j\infty$ (no sheet to a perfectly conducting sheet), the cutoff numbers will vary between the two curves shown. This variation is summarized in Table I., the cutoff values are computed for some of the lower order modes. As the lossless sheet values are decreased from zero, the step-index cutoff values increase towards the cutoff values of the circular, metallic guide.

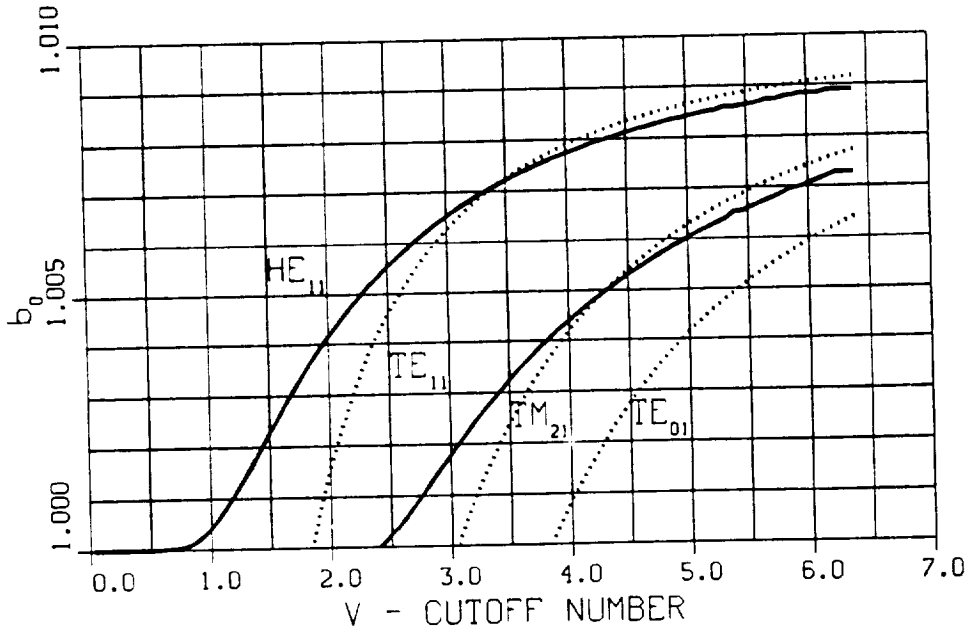

Figure 4. Normalized propagation constant vs. cutoff number, step-index fiber ——— and circular, metallic guide ..... .

TABLE I. CUTOFF VALUES FOR LOSSLESS SHEETS
$u = V$ At Cutoff

η_{01}, η_{02}, α	HE_{11}	HE_{21}	TE_{01}
-j0. ,-j0. , 0	0	2.41	2.41
-j.2 ,-j.1 , 60	.75	2.58	3.06
-j.2 ,-j.1 , 45	.86	2.71	3.14
-j.2 ,-j.1 , 0	1.02	2.84	3.26
-j.3 ,-j.2 , 0	1.24	2.95	3.41
-j2 , -j1 , 0	1.74	3.05	3.76
-j∞ ,-j∞ , 0	1.84	3.05	3.83

3.2 Lossy Sheet Parameters

Suppose the sheet admittances are real or complex. Because loss terms are now present, the propagation constant will no longer be real. Instead,

(41) $$b_0 = b_{0r} + jb_{0i}.$$

The field equations will contain the term

(42) $$e^{b_{0i}z}e^{-jb_{0r}z},$$

where b_{0i} is the normalized attenuation constant. Therefore, solutions with $b_{0i} < 0$ will describe attenuated, propagating modes.

The remaining graphs are plots of both the real and imaginary parts of the normalized propagation constant versus normalized radius. The normalized attenuation constants have been plotted up to the cutoff values of the propagation constant. Figure 5 represents the propagation curves for the TM_{01} mode for sheet admittances that are real. The solid curve is the step-index fiber plot. Although the cutoff radius remained unchanged in the lossless examples, it has been increased for sheet parameters with small losses. For larger losses, the TM_{01} mode cutoff radius again becomes that of the step-index fiber.

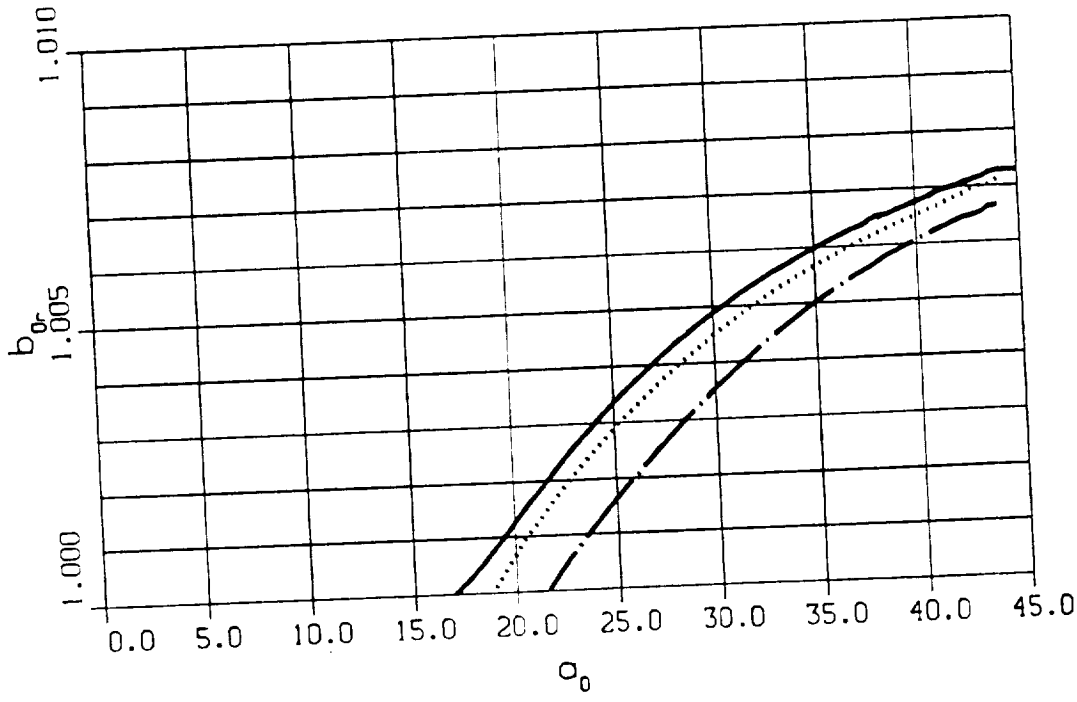

Figure 5. Normalized propagation constant versus normalized radius for TM_{01} mode with real sheet admittances.

η_{01}	η_{02}	α	
0	0	0	———
.1	.1	0	.....
.2	.2	0	·——·——·
1	1	0	———

The corresponding attenuation curves in Figure 6 reveal that a large attenuation near cutoff occurs for $\eta_{01} = -j.2$ and becomes negligible near cutoff for $\eta_{01} = -j1$. The propagation constant curve for the $\eta_{01} = -j1$ case is not distinguishable from that of the step-index fiber.

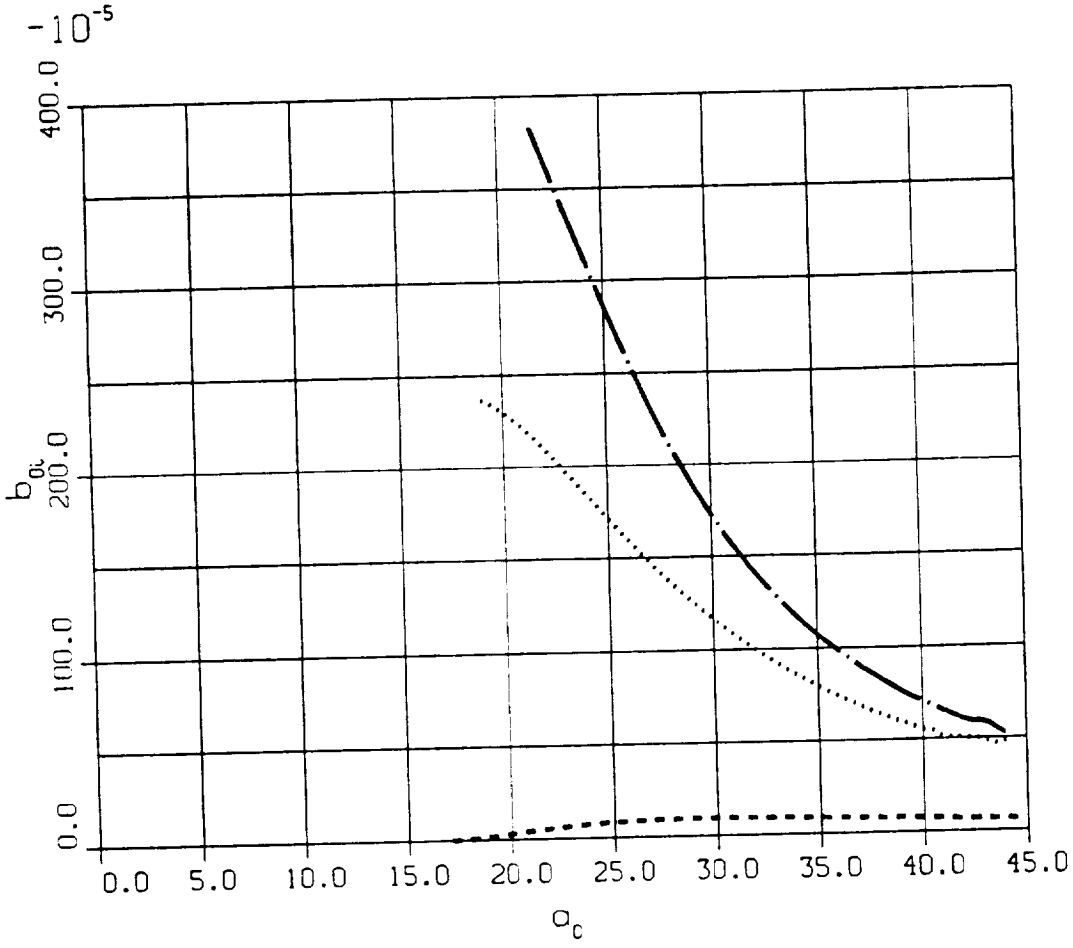

Figure 6. Normalized attenuation constant versus normalized radius for TM_{01} mode with real sheet admittances.

η_{01}	η_{02}	α	
.1	.1	0	.....
.2	.2	0	·——·
1	1	0	- - - -

The same TM_{01} mode was examined for complex sheet parameters. While the propagation curves of Figure 7 are now nearly coincident, the attenuation near cutoff has been reduced significantly (Figure 8).

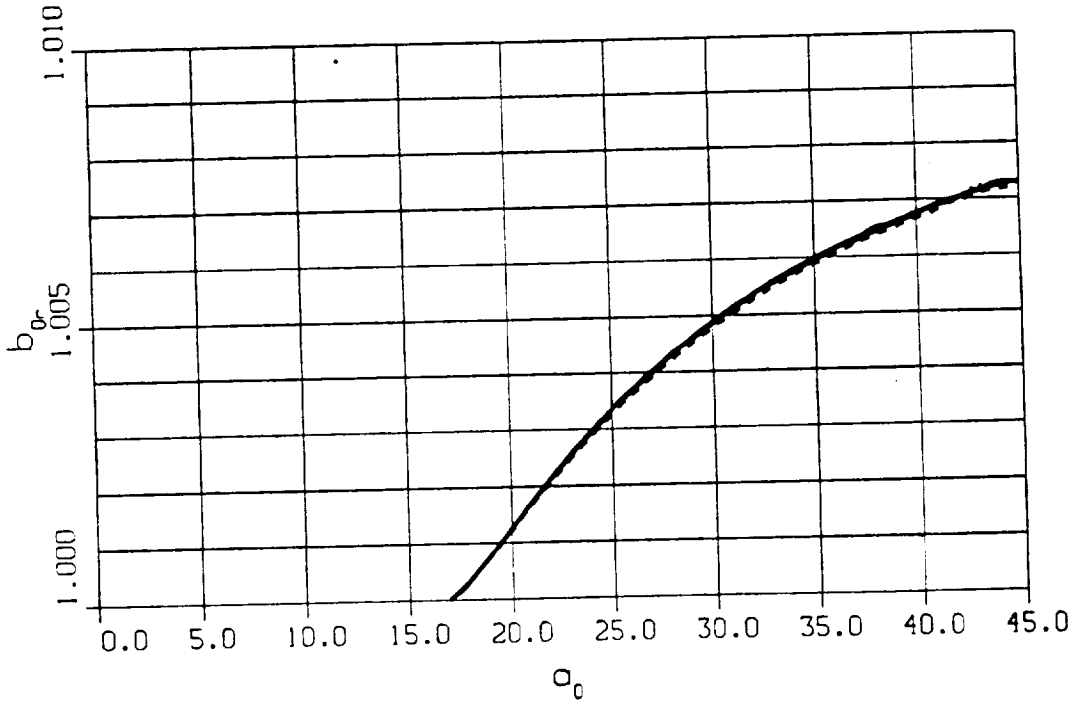

Figure 7. Normalized propagation constant versus normalized radius for TM_{01} mode with complex sheet admittances.

η_{01}	η_{02}	α	
0	0	0	———
.1-j.2	.1-j.1	0	·····
.2-j.3	.2-j.2	0	·———·
1-j2	1-j1	0	– – – –

(all four curves are nearly coincident)

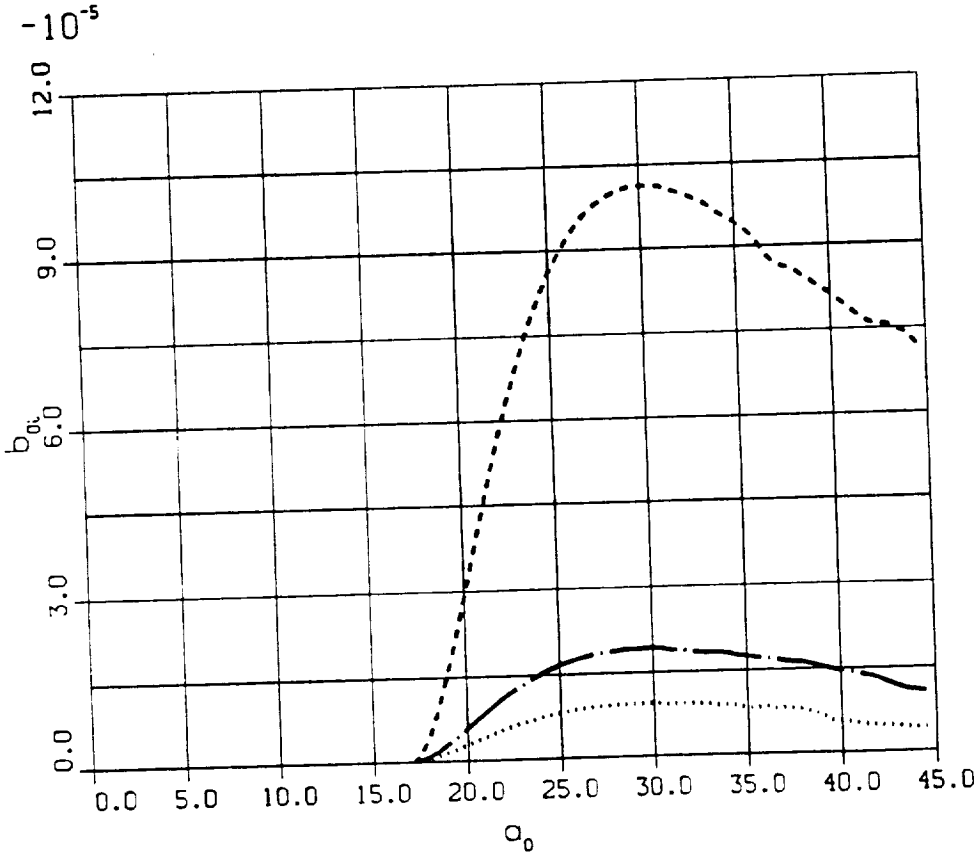

Figure 8. Normalized attenuation constant versus normalized radius for TM_{01} mode with complex sheet admittances.

η_{01}	η_{02}	α	
.1-j.2	.1-j.1	0	.....
.2-j.3	.2-j.2	0	·——·
1-j2	1-j1	0	– – – –

The effect of real sheet admittances on the HE_{11} mode is now examined. Figure 9 shows the expected rise in cutoff radius.

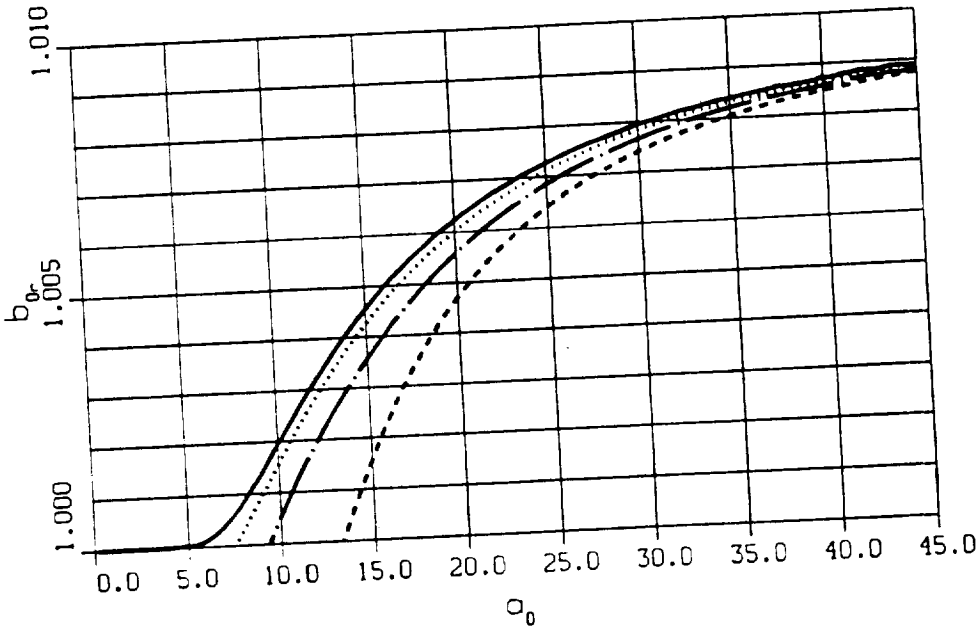

Figure 9. Normalized propagation constant versus normalized radius for HE_{11} mode with real sheet admittances.

η_{01}	η_{02}	α	
0	0	0	———
.1	.1	0	.....
.2	.2	0	·———·
1	1	0	– – – –

Note the slight change in scale of the attenuation curve of Figure 10. The lower values of η_{01} show less attenuation near cutoff than the TM_{01} mode. However, for the $\eta_{01} = -j1$, the HE_{11} mode has the greater attenuation.

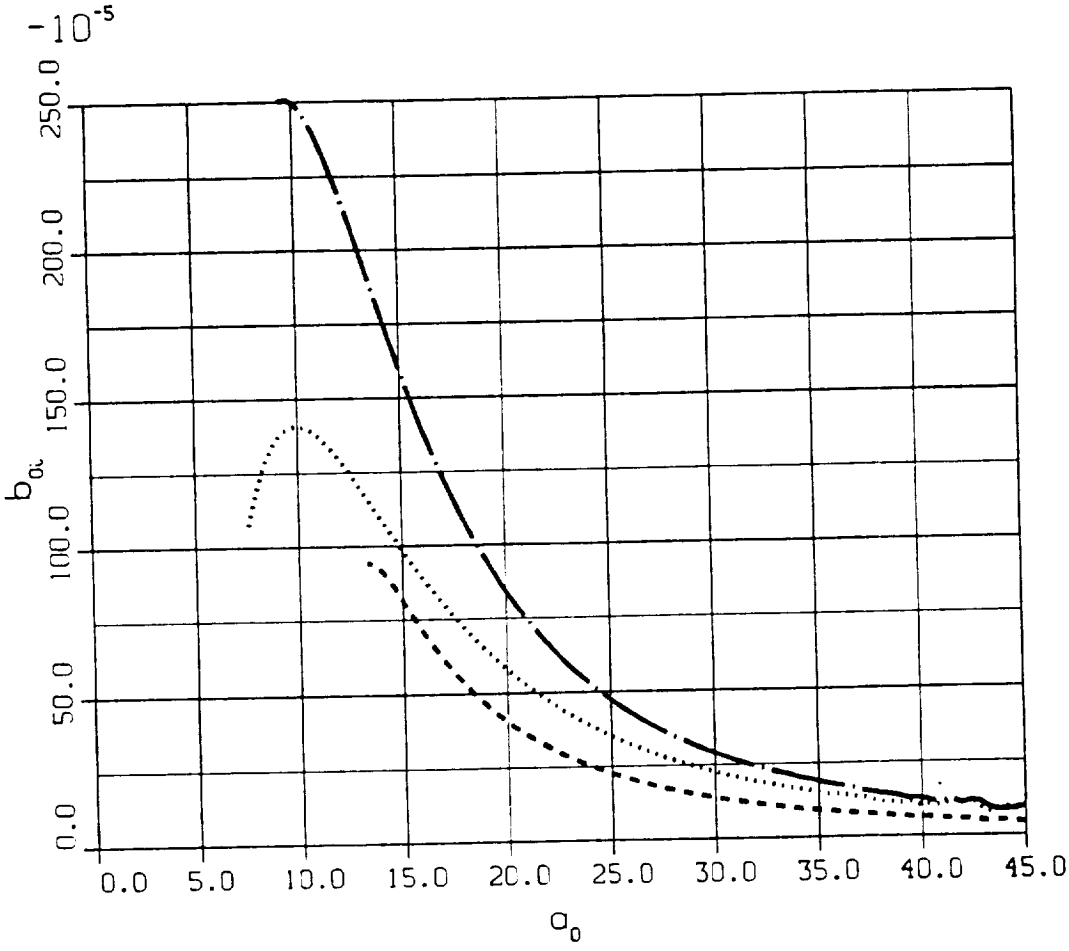

Figure 10. Normalized attenuation constant versus normalized radius for HE_{11} mode with real sheet admittances.

η_{01}	η_{02}	α	
.1	.1	0	.....
.2	.2	0	·——·
1	1	0	– – – –

Figure 11 shows again how the complex sheet parameters tend to move the propagation curves towards the metallic guide limit.

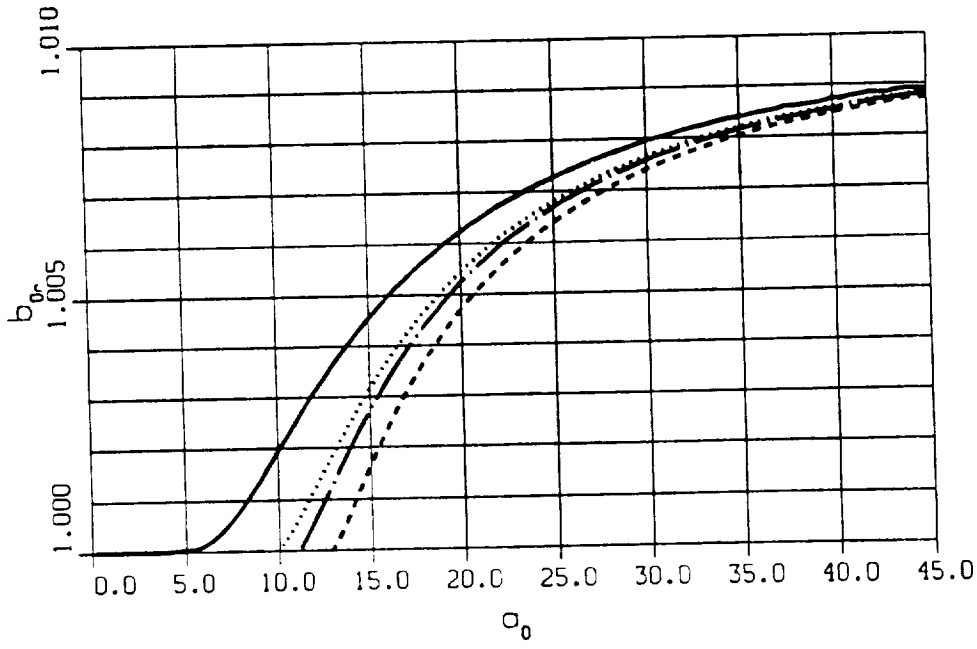

Figure 11. Normalized propagation constant versus normalized radius for HE_{11} mode with complex sheet admittances.

η_{01}	η_{02}	α	
0	0	0	———
.1-j.2	.1-j.1	0	·····
.2-j.3	.2-j.2	0	·———
1-j2	1-j1	0	– – – –

Figure 12 shows that the complex admittances also reduce the attenuation of the HE_{11} mode, but not as large a reduction as that which resulted for the TM_{01} mode.

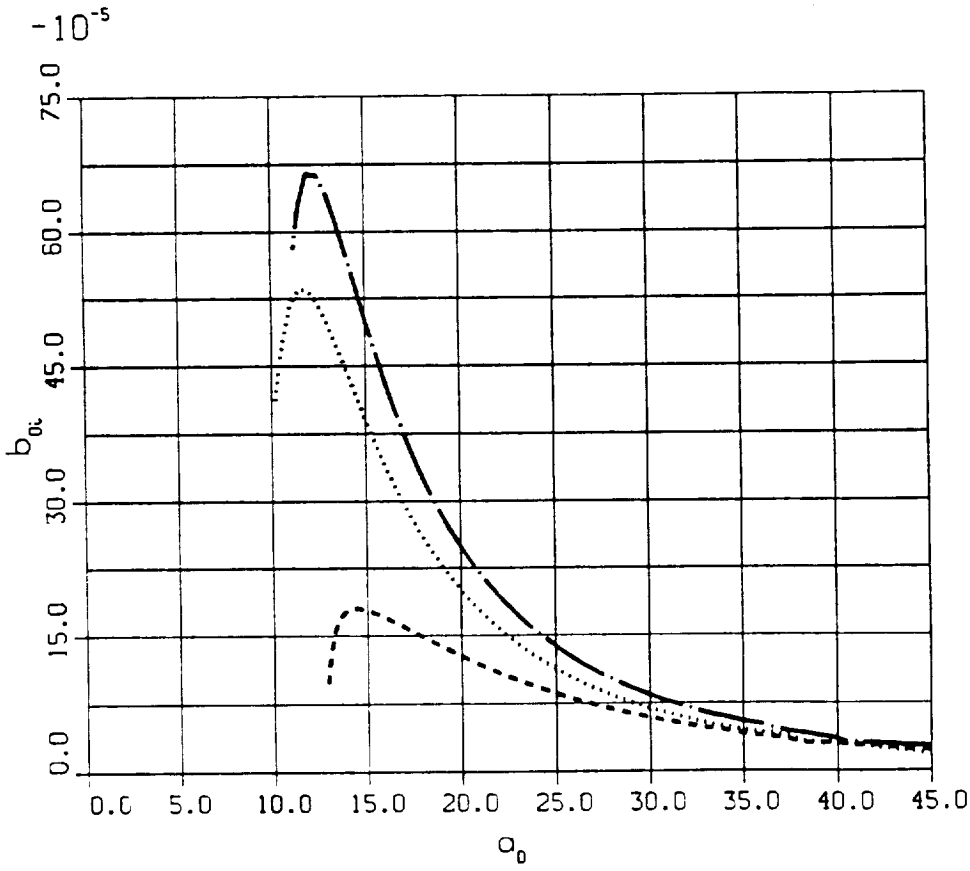

Figure 12. Normalized attenuation constant versus normalized radius for HE_{11} mode with complex sheet admittances.

η_{01}	η_{02}	α	
.1-j.2	.1-j.1	0	.....
.2-j.3	.2-j.2	0	.——.
1-j2	1-j1	0	- - - -

Finally, Figures 13 and 14 reveal the dependence of the propagation propagation and attenuation curves on the sheet orientation. For the HE_{11} mode, increasing the parameter α has little effect on the cutoff radius, while slightly increasing the attenuation near cutoff.

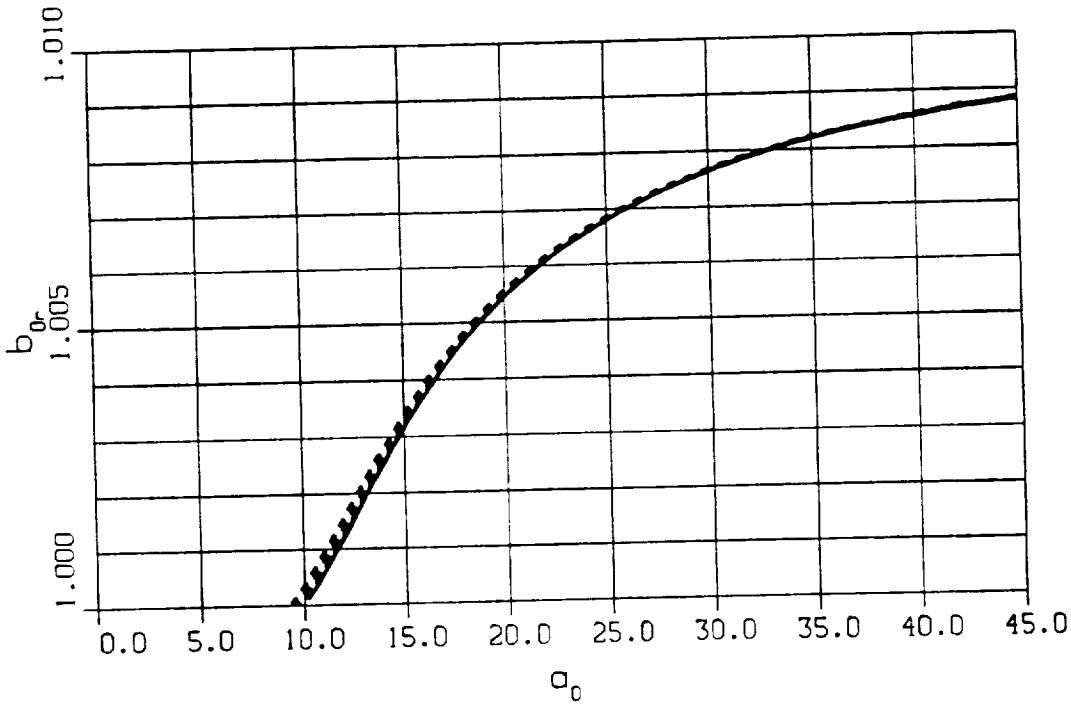

Figure 13. Normalized propagation constant versus normalized radius for HE_{11} mode with complex sheet admittances and variation in the parameter α.

η_{01}	η_{02}	α	
.1-j.2	.1-j.1	0	———
.1-j.2	.1-j.1	30	·····
.1-j.2	.1-j.1	45	– – – –
.1-j.2	.1-j.1	60	·———·

(all four curves nearly coincident)

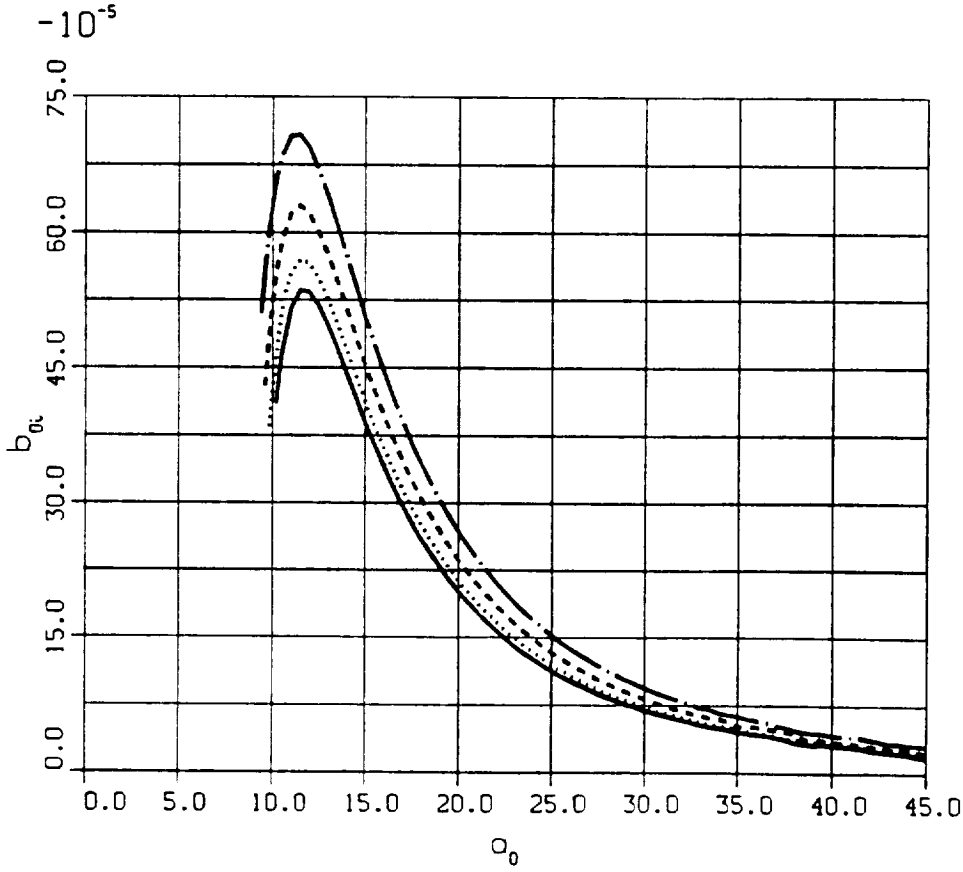

Figure 14. Normalized attenuation constant versus normalized radius for HE_{11} mode with complex sheet admittances and variation in the parameter α.

η_{01}	η_{02}	α	
.1-j.2	.1-j.1	0	———
.1-j.2	.1-j.1	30	·····
.1-j.2	.1-j.1	45	– – – –
.1-j.2	.1-j.1	60	·———·

The general effect of the anisotropic sheet admittance is to raise the cutoff frequencies of the propagating modes. The cutoff frequncies vary from those of the step-index fiber to those of the circular, metallic guide as the sheet admittance varies from zero to infinity. Use of lossy sheets also affected the cutoff of the lower order TM modes which were unchanged in the lossless case. Attenuated, propagating modes exist when lossy sheet admittances are present, although it was shown that the attenuation could be controlled by adjusting the sheet parameters and orientation.

The dielectric waveguide analyzed is a relatively simple structure. However, the method of analysis used is easily extended to multilayered structures. Additional boundary conditions will result, the jump immitance condition is applied at each sheet interface. The method of solution will remain the same; solutions must be found for a complex valued dispersion relation.

APPENDICES

APPENDIX A. DISPERSION RELATION ROOT COMPUTATION

As stated previously, the dispersion relation, (30), is quadratic in $\mathcal{J}_m$. It is of the form

$$C_0\mathcal{J}_m^2 + C_1\mathcal{J}_m + C_2 = 0 \tag{A1}$$

with,

$$C_0 = \epsilon_r\mu_r(1 - j\eta_1 a_0\mathcal{K}_m) \tag{A2a}$$

$$C_1 = \left\{-j\eta_1\mu_r a_0\mathcal{K}_m^2 + \mathcal{K}_m[\mu_r(1+\eta_1\eta_2-\eta_3^2)+\epsilon_r] + \frac{j\mu_r}{a_0}\left[\left(\frac{mb_0a_0}{q^2}\right)^2\eta_1 - 2\frac{mb_0a_0}{q^2}\eta_3 + \eta_2\right]\right\} \tag{A2b}$$

$$C_2 = \mathcal{K}_m^2 + \frac{j\mathcal{K}_m}{a_0}\left[\left(\frac{mb_0a_0}{u^2}\right)^2\eta_1 + 2\frac{mb_0a_0}{u^2}\eta_3 + \eta_2\right] - (mb_0)^2\left(\frac{1}{u^2}+\frac{1}{q^2}\right)^2 = 0 \quad . \tag{A2c}$$

By applying the quadratic formula, solutions for H-type and E-type modes are easily separated [3]:

$$F(a_0, b_0) = 2C_0\mathcal{J}_m + C_1 \pm \sqrt{C_1^2 - 4C_0C_2} = 0. \tag{A3}$$

The HE modes are the solutions taken from the positive radical and the EH modes are taken from the negative sign [4]. The method of solution is the Newton-Raphson technique; it is capable of determining complex roots [13] and thus accommodates both the lossless and lossy cases.The dispersion relation shown in (A3) is a function of two variables. The following FORTRAN program solves this dispersion relation for selected parameters of the dielectric core and sheet admittances ($\epsilon_r, \mu_r, \eta_{01}, \eta_{02}$, and α). Initial guesses for the Newton-Raphson technique are provided from a knowledge of the cutoff conditions or from published data for the step-index fiber [2]. The calculated root is then used as the next guess. As one of the variables is incremented, this is continued to generate

APPENDIX A. (Continued)

the dispersion and attenuation plots over the desired range. Cutoff values are obtained in a similar fashion, the cutoff equations are substituted in the function subroutine section of the program.

```
C*************************************************************************
C                                                                        *
C                        PROGRAM DISP                                    *
C                                                                        *
C   THIS PROGRAM CACULATES THE ZEROS OF THE DISPERSION RELATION FOR A    *
C   CYLINDRICAL WAVEGUIDE CONSISTING OF A DIELECTRIC CORE SURROUNDED     *
C   BY AN ANISOTROPIC SHEET IN FREE SPACE.                               *
C                                                                        *
C                                                                        *
C   VARIABLE ASSIGNMENTS                                                 *
C     BO    -  NORMALIZED PROPAGATION CONSTANT                           *
C     KTO   -  NORMALIZED TRANVERSE WAVE NUMBER INSIDE CORE              *
C     GTO   -  NORMALIZED TRANSVESE WAVE NUMBER OUTSIDE CORE             *
C     AO    -  NORMALIZED RADIUS                                         *
C     U     =  KTO*AO, BESSEL ARGUMENT                                   *
C     Q     =  GTO*AO, MODIFIED BESSEL ARGUMENT                          *
C     BGES  -  INITIAL GUESS FOR BO                                      *
C     AGES  -  INITIAL GUESS FOR AO                                      *
C     FOUT  -  VALUE OF DISPERSION RELATION                              *
C                                                                        *
C     MSTART,MSTOP- RANGE OF BESSEL ORDER                                *
C     HSTART,HSTOP- RANGE OF MODE TYPE (HE=0, EH=1)                      *
C                                                                        *
C     EPSR  -  CORE RELATIVE PERMITTIVITY                                *
C     MUR   -  CORE RELATIVE PERMEABILITY                                *
C                                                                        *
C     NO1,NO2,ANG-  SHEET PARAMETERS                                     *
C     N1,N2,N3    -  TRANSFORMED SHEET PARAMETERS                        *
C                                                                        *
C*************************************************************************
C
C
C-------------------------------------------------------------------------
C COMMON BLOCK PARAM PASSES BACK MEDIA DATA AND TRANSVERSE WAVE NOS.
C BETWEEN SUBROUTINES.
C
      COMMON /PARAM/ U,KTO,Q,GTO,NO1,NO2,ANG,EPSR,MUR,ITS2,NIN2
      COMPLEX U,KTO,Q,GTO,NO1,NO2
      REAL ANG,EPSR,MUR,R
      INTEGER ITS2,NIN2
C-------------------------------------------------------------------------
      INTEGER MSTART,MSTOP,M,HSTART,HSTOP,H,ITS,INC2,AMIN
      REAL AO,AGES,REF,DEC
      COMPLEX BO,BGES,FOUT,F
      EXTERNAL NEWT,CBJNS
C
    3 FORMAT (1X,I2)
C
C-------------------------------------------------------------------------
C     READ IN DATA AND INIT GUESS
C
        OPEN(UNIT=79,STATUS='OLD')                  DIELECTRIC PARAMETERS
C
        READ(79,7)EPSR,MUR
    7   FORMAT(1X,2(F8.4,2X))                       SHEET PARAMETERS
C
        READ(79,9)NO1,NO2,ANG
    9   FORMAT(1X,5(F8.4,2X))                       MIN RADIUS,MODE ORDER/TYPE
C
        READ(79,11)AMIN,MSTART,MSTOP,HSTART,HSTOP
   11   FORMAT(1X,5(I4,2X))                         INIT. VALUES AND ITERATIONS
C
        READ(79,13)BGES,AGES,ITS
   13   FORMAT(1X,3(F10.5,2X),I3)
```

APPENDIX A. (Continued)

```
      CLOSE(UNIT=79)
C
C   OPEN OUTPUT FILE
C
      OPEN(UNIT=80)
      WRITE(80,*)(' EPSR              MUR')
      WRITE(80,7)EPSR,MUR
      WRITE(80,*)(' NO1               NO2                    ANG')
      WRITE(80,9)NO1,NO2,ANG
      WRITE(80,*)(' MIN RAD    MSTART   MSTOP   HSTART   HSTOP')
      WRITE(80,11)AMIN,MSTART,MSTOP,HSTART,HSTOP
      WRITE(80,*)(' BO GUESS          AO GUESS               ITERATIONS')
      WRITE(80,13)BGES,AGES,ITS
C
C-----------------------------------------------------------------------
C***********************************************************************
C   CALCULATE ZEROS OF DISPERSION RELATION
C***********************************************************************
      WRITE(6,*)('DO YOU WANT ITERATIONS PRINTED?')
      WRITE(6,*)('REAL ROOTS: Y(1) N(2)   COMPLEX:Y(-1),N(-2)')
      READ(6,3)NIN2
      WRITE(6,*)('INPUT RADIUS INCREMENT?')
      WRITE(6,*)('DEC=IDEC*.1')
      READ(6,3)IDEC
      REF=SQRT(EPSR*MUR)
C
      WRITE(80,*)(' MODE TYPE')
      WRITE(80,*)('H='),H
      DO 1 H=HSTART,HSTOP
        DO 2 M=MSTART,MSTOP
          BO=BGES
          AO=AGES
          R=FLOAT(AMIN)
          DEC=FLOAT(IDEC)*.1
          ITS2=ITS
          WRITE(80,*)(' ROOT FILE')
          WRITE(80,*)('M='),M
          WRITE(80,*)(' AO    BO          U           Q          FOUT')
          WRITE(80,39)
   39     FORMAT (3(/))
C
C DO LOOP SEARCHES FOR ROOTS WITHIN THE BOUNDS OF BO FOR
C PROPAGATING MODES
C
        DO 4 WHILE ((REAL(BO) .GE. 1.)
     &         .AND.  (REAL(BO) .LE. REF)
     &         .AND.  (AO .GT. R))
          CALL NEWT(BO,AO,M,H,FOUT)
          WRITE(6,*)AO,BO,U
          WRITE(80,14)AO,BO,U,FOUT
   14     FORMAT(1X,2(F10.5),2X,E12.5,4(F8.4))
          AO=AO-DEC
    4   ENDDO
    2   CONTINUE
    1 CONTINUE
      CLOSE(UNIT=80)
      STOP
   99 END
C
C
C***********************************************************************
C COMPLEX FUNCTION F CALCULATES THE VALUE OF THE DISPERSION RELATION
C***********************************************************************
C
C
      COMPLEX FUNCTION F(BO,AO,M,H)
C-----------------------------------------------------------------------
C COMMON BLOCK PARAM PASSES BACK MEDIA DATA AND TRANSVERSE WAVE NOS.
C BETWEEN SUBROUTINES.
C
```

APPENDIX A. (Continued)

```
      COMMON /PARAM/ U,KTO,Q,GTO,NO1,NO2,ANG,EPSR,MUR,ITS2,NIN2
      COMPLEX U,KTO,Q,GTO,NO1,NO2
      REAL ANG,EPSR,MUR,R
      INTEGER ITS2,NIN2
C-----------------------------------------------------------------------
C-----------------------------------------------------------------------
CC
CC   FUNCTION F VARIABLES
CC     FJ    -  STORES VALUES OF BESS FUNC
CC     FK    -  STORES VALUES OF MOD BESS FUNC
CC     JM    -  DERIVATIVE OF BESS FUNC / (BESS ARG * BESS FUNC.)
CC     KM    -  ABOVE RATIO FOR MOD BESS FUNC
CC
CC     CO,C1,C2 - COEFFICIENTS OF QUDRATIC FORM OF DISPERSION RELATION
CC     DISCR    - QUADRATIC FORM DISCRIMINANT
CC
C-----------------------------------------------------------------------
C
      INTEGER H
      REAL AO,RANG
      COMPLEX N1,N2,N3
      COMPLEX BO,JM,KM,MB,CO,C1,C2,DISCR,FK(5),FJ(5),CJ
      PI=DCONST('PI')
      RANG=ANG*PI/180.
      N1=NO1*COS(RANG)**2+NO2*SIN(RANG)**2
      N2=NO1*SIN(RANG)**2+NO2*COS(RANG)**2
      N3=(NO1-NO2)*COS(RANG)*SIN(RANG)
      KTO=CSQRT(EPSR*MUR*(1.,0.)-BO**2)
      GTO=CSQRT(BO**2-(1.,0.))
      U=KTO*AO
      Q=GTO*AO
      CALL CBJNS(U,M+2,FJ)
      CALL CBESNK(GA,M+2,FK)
      IF (M .EQ. 0) THEN
        JM=(-1.,0.)*FJ(M+2)/FJ(M+1)/U
        KM=-FK(M+2)/FK(M+1)/Q
      ELSE
        JM=(FJ(M)-FJ(M+2))/FJ(M+1)/U
        KM=-(FK(M)+FK(M+2))/FK(M+1)/Q
      ENDIF
      CJ=(0.,1.)
      MB=FLOAT(M)*B
      CO=EPSR*MUR*(1.-CJ*N1*AO*KM)
      C1=-CJ*N1*MUR*AO*KM**2+KM*(MUR*(1.+N1*N2-N3**2)+EPSR)
     &   +CJ*MUR*((MB*AO/Q**2)**2*N1-2.*MB*AO*N3/Q**2+N2)/AO
      C2=KM**2+CJ*KM*((MB*AO/U**2)**2*N1+2.*MB*AO*N3/U**2+N2)/AO
     &   -(MB*(1/U**2+1/Q**2))**2
      DISCR=CSQRT(C1**2-4.*CO*C2)
      IF (H .EQ. 1) THEN
        DISCR=-DISCR
      ENDIF
      F=JM+(C1+DISCR)/(2.*CO)
      RETURN
      END
C
CC
CC
CC
C*********************************************************************
C  SUBROUTINE NEWT PERFORMS NEWT-RAPHSON ITERATIVE PROCEDURE TO
C  CALC ROOTS OF DIPERSION RELATION
C*********************************************************************
C
      SUBROUTINE NEWT(BO,AO,M,H,FOUT)
C-----------------------------------------------------------------------
C COMMON BLOCK PARAM PASSES BACK MEDIA DATA AND TRANSVERSE WAVE NOS.
C BETWEEN SUBROUTINES.
C
      COMMON /PARAM/ U,KTO,Q,GTO,NO1,NO2,ANG,EPSR,MUR,ITS2,NIN2
      COMPLEX U,KTO,Q,GTO,NO1,NO2
```

APPENDIX A. (Continued)

```
      REAL ANG,EPSR,MUR,R
      INTEGER ITS2,NIN2
C-----------------------------------------------------------------------
C-----------------------------------------------------------------------
C
C  SUBROUTINE NEWT VARIABLES
C    FP  -  APPROXIMATES DERIVATIVE OF DISPERSION RELATION OVER
C           THE INTERVAL INC IN BO
C
C-----------------------------------------------------------------------
      INTEGER M,H
      REAL AO
      COMPLEX BO,FOUT,F,FP,INC
      INC=(.10E-06,0.)
      IF (NIN2 .LT. 0) THEN
      INC=INC*(1.,1.)
      ENDIF
      I=1
      DO 12 WHILE ((REAL(BO) .GE. 1.)
     &      .AND.  (REAL(BO) .LE. (SQRT(EPSR*MUR)))
     &      .AND.  (I .LT. ITS2))
        FP=(F(BO+INC,AO,M,H)-F(BO,AO,M,H))/INC
        BO=BO-(F(BO,AO,M,H)/FP)
        FOUT=F(BO,AO,M,H)
        IF (ABS(NIN2) .EQ. 1) THEN
        WRITE(6,21)AO,BO,U,Q,FOUT
21      FORMAT(1X,9(F8.4,2X))
        ENDIF
        I=I+1
12    ENDDO
      RETURN
      END
```

APPENDIX B. DERIVATION OF CUTOFF CONDITIONS

B.1 Limits at Cutoff

To find the cutoff conditions of the various modes, the dispersion relation

$$
\begin{aligned}
&\mathcal{J}_m^2\epsilon_r\mu_r(1-j\eta_1 a_0\mathcal{K}_m)\\
&+\mathcal{J}_m\left\{-j\eta_1\mu_r a_0\mathcal{K}_m^2+\mathcal{K}_m[\mu_r(1+\eta_1\eta_2-\eta_3^2)+\epsilon_r]\right.\\
&\qquad\left.+\frac{j\mu_r}{a_0}\left[\left(\frac{mb_0a_0}{q^2}\right)^2\eta_1-2\frac{mb_0a_0}{q^2}\eta_3+\eta_2\right]\right\}\\
&+\mathcal{K}_m^2+\frac{j\mathcal{K}_m}{a_0}\left[\left(\frac{mb_0a_0}{u^2}\right)\eta_1+2\frac{mb_0a_0}{u^2}\eta_3+\eta_2\right]-(mb_0)^2\left(\frac{1}{u^2}+\frac{1}{q^2}\right)^2=0 \quad .
\end{aligned}
\tag{B1}
$$

is examined in the limit as

$$q \to 0 \tag{B2a}$$

$$b \to 1 \tag{B2b}$$

$$u \to a_0\sqrt{\epsilon_r\mu_r-1} \quad . \tag{B2c}$$

Under these conditions, $\mathcal{K}_m$ takes the following form [4]:

$$m=0; \quad \mathcal{K}_0 \to \frac{1}{q^2 ln\frac{\Gamma q}{2}} \tag{B3a}$$

$$m=1; \quad \mathcal{K}_1 \to -\frac{1}{q^2}+\frac{1}{2}ln\frac{q}{2} \tag{B3b}$$

$$m>1; \quad \mathcal{K}_m \to -\frac{m}{q^2}-\frac{1}{2(m-1)} \quad . \tag{B3c}$$

B.2 Case I. : $m>1$

Equation (B1) is multiplied out and the factors of q are collected. Note that

$$\left(\frac{1}{u^2}+\frac{1}{q^2}\right)^2=\frac{1}{q^4}+\frac{2}{u^2q^2}+\frac{1}{u^4} \tag{B4a}$$

$$\mathcal{K}_m^2=\frac{m^2}{q^4}+\frac{m}{(m-1)q^2}+\frac{1}{4(m-1)^2} \quad . \tag{B4b}$$

APPENDIX B. (Continued)

The dispersion relation will be of the form $X/q^2 + Y$. Therefore, constant terms with no q dependence will be dropped. They will approach zero as the equation is multiplied by q^2 and q approaches zero. The surviving terms are

$$
\begin{aligned}
&\mathcal{J}_m^2(\frac{1}{q^2})(j\eta_1\epsilon_r\mu_r a_0 m)+ \\
&\mathcal{J}_m(\frac{1}{q^4})[j\eta_1\mu_r a_0 m^2(b_0^2-1)] \\
&\mathcal{J}_m(\frac{1}{q^2})[-j\eta_1\mu_r a_0\frac{m}{m-1} - 2j\eta_3\mu_r m b_0 - m(\mu_r(1+\eta_1\eta_2-\eta_3^2)+\epsilon_r)] \\
&-(\frac{1}{q^4})(b_0^2-1) - (\frac{1}{q^2})\left[\frac{m}{m-1} - 2(\frac{mb_0}{u})^2 - j\frac{m}{a_0}[(\frac{mb_0a_0}{u^2})^2\eta_1 + 2\frac{mb_0a_0}{u^2}\eta_3+\eta_2]\right] .
\end{aligned}
\tag{B5}
$$

Since $a_0^2(b_0^2-1) = q^2$,(B5) actually contains terms of highest order $\frac{1}{q^2}$,

$$
\begin{aligned}
&\mathcal{J}_m^2(\frac{m}{q^2})j\eta_1\epsilon_r\mu_r a_0 \\
&\mathcal{J}_m(\frac{m}{q^2})\left[j\eta_1\mu_r\left(\frac{m}{a_0}-\frac{a_0}{m-1}\right) - 2j\eta_3\mu_r - \mu_r(1+\eta_1\eta_2-\eta_3^2) - \epsilon_r\right] \\
&-\frac{m}{q^2a_0}\left(\frac{m}{a_0}-\frac{a_0}{m-1}\right) - \frac{2m}{u^2} - \frac{j}{a_0}\left((\frac{ma_0}{u^2})^2\eta_1 + 2\frac{ma_0}{u^2}\eta_3+\eta_2\right) = 0 .
\end{aligned}
\tag{B6}
$$

Multilplying (B6) by q^2/m , the result is,

$$
\begin{aligned}
&\mathcal{J}_m^2 j\eta_1\epsilon_r\mu_r a_0 \\
&\mathcal{J}_m\left[j\eta_1\mu_r\left(\frac{m}{a_0}-\frac{a_0}{m-1}\right) - 2j\eta_3\mu_r - \mu_r(1+\eta_1\eta_2-\eta_3^2) - \epsilon_r\right] \\
&-\frac{1}{a_0}\left(\frac{m}{a_0}-\frac{a_0}{m-1}\right) - \frac{2m}{u^2} - \frac{j}{a_0}\left((\frac{ma_0}{u^2})^2\eta_1 + 2\frac{ma_0}{u^2}\eta_3+\eta_2\right) = 0 ,
\end{aligned}
\tag{B7}
$$

which is equivalent to (37), the cutoff equation for the $m > 1$ modes.

APPENDIX B. (Continued)

B.3 Case II. : $m = 1$

For the $m = 1$ cutoff conditions, use $\mathcal{K}_1$ from (B3b) and

$$\mathcal{K}_1^2 = \frac{1}{q^4} - \frac{ln\frac{q}{2}}{q^2} + (ln\frac{q}{2})^2. \tag{B8}$$

(B1) is multiplied by $\frac{q^2}{ln\frac{q}{2}}$, this allows removal of the terms containing $(\mathcal{K}_m)^1$,terms up to the order $\frac{1}{q^2}$, and constants with respect to q. The surviving terms are

$$(\frac{q^2}{ln\frac{q}{2}})[-j\eta_1\mu_r a_0\mathcal{J}_m + 1][\mathcal{K}_m^2 - m^2 b_0^2] = 0. \tag{B9}$$

With applications of De l'Hospital's Rule, this is equivalent to

$$-j\eta_1\mu_r a_0\mathcal{J}_1 + 1 = 0. \tag{B10}$$

Equation (B10) is the same as (39), the cutoff equation for the $m = 1$ modes.

B.4 Case III. : $m = 0$

Letting $m = 0$, the remaining terms of (B1) are,

$$\begin{aligned}&\mathcal{J}_0^2\epsilon_r\mu_r(1 - j\eta_1 a_0\mathcal{K}_0)\\ &+\mathcal{J}_0\Big\{-j\eta_1\mu_r a_0\mathcal{K}_0^2 + \mathcal{K}_0[\mu_r(1 + \eta_1\eta_2 - \eta_3^2) + \epsilon_r] + \frac{j\mu_r\eta_2}{a_0}\Big\}\\ &+\mathcal{K}_0^2 + \frac{j\mathcal{K}_0\eta_2}{a_0} = 0 \quad .\end{aligned} \tag{B11}$$

This equation actually factors into the following two solutions

$$\text{TEcase} \qquad \mathcal{J}_0 = \frac{\mathcal{K}_0(\epsilon_r - \eta_3^2)}{\epsilon_r\mu_r(1 - j\eta_1 a_0\mathcal{K}_0} \tag{B12a}$$

$$\text{TMcase} \qquad \mathcal{J}_0 = \frac{-1}{\epsilon_r}(\mathcal{K}_0 + j\frac{\eta_2}{a_0}). \tag{B12b}$$

As $\mathcal{K}_0$ becomes large at cutoff, these reduce to the quantities in (40) by inspection,

$$\text{TM}_{0n}\text{ modes :} \qquad J_0(u) = 0 \tag{B13a}$$

$$\text{TE}_{0n}\text{ modes :} \qquad \frac{J_0'(u)}{uJ_0(u)} = \frac{\epsilon_r - \eta_3^2}{j\eta_1 a_0\epsilon_r\mu_r} \quad . \tag{B13b}$$

From equation (B12b), it is apparent cutoff frequencies of the lossless TM_{01} modes are not affected by sheet parameters.

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